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STUDY OF GRANULATION BY MEANS OF A ROTATING DRIER WITH PRACTICAL APPLICATION TO THE DESIGNING OF AN APPARATUS AND PRODUCTION CONTROL

by E. J. Leger.

FOREWORD

The appearance on the fertiliser market of a granulated material of a definite range of particle size in substitution of a powdered material goes back to several years before the war. This innovation was brought about by the desire to provide customers, who became increasingly exacting, with qualities which were constantly being improved. Originally, the aim was of a purely commercial nature, and the endeavour, to supply farmers with a fertiliser of better storage quality, which could be spread more readily, met with complete success after some initial vicissitudes. Later, agricultural circles were induced to study this new physical aspect as compared with the old one from the point of view of fertilising value. Statistics were established, and the research is still going on in many countries. If there is apparently no unanimous support in favour of the agricultural supremacy of granulated fertilisers,as compared with powdered fertilisers, it is probably because the question is a complicated one; it requires a great deal of observation, spread over a number of years and, finally, the advantage of one form over another is perhaps not very marked.

On the other hand, it is indisputable that the granulated form greatly facilitates storage of the fertiliser during the period in between the agricultural campaigns; it eliminates to a very large extent the enormous difficulties encountered when moving the material for shipment; it almost completely eliminates the well-known phenomenon of caking and, above all, it renders the spreading on the soil by means of a modern drill easier, quicker, and more precise.

We leave to others, who are greater authorities, the task of voicing an opinion on the fertiliser qualities of a granulated fertiliser, taking only into account the favour which is constantly shown by the farmer, who definitely has the last word. Let us bear in mind, as clear evidence, the growing demand for granulated products, and the recent application of the granulation process to superphosphate itself.

Confronted with this situation, the rôle of the industrialist has been to adapt himself to circumstances; all the more so as he was interested in handling in his installations a readily manageable material instead of compact and hard lumps, the removal of which requires a good deal of work and money. As so often happens with sudden developments, the industrialist must act quickly if his market position is not to suffer but, if possible, to be improved.

Plants were constructed and processes developed which were successful to a varying degree. The child came into the world fifteen years ago, and to-day it is growing fast. The installations multiply and take on various shapes, each type being recommended on account of some special feature. Actually, the different processes each suit a specific case, and the determining factor in the choice of the type is to be sought in reasons of a widely divergent character. The nature of the product to be treated is obviously the primary determining factor. The readiness with which ammonium nitrate assumes a divided state is not the same as that of superphosphate, and it is understandable that an apparatus suitable for one is unsuitable for the other. These are the technical reasons. Secondly, there are reasons of expediency. The chemical improvement of a fertiliser, for example, may be brought about by varied processes, and it is evident that an industrialist, who wishes to produce a granulated fertiliser, is compelled to fit the granulation phase into the actual production operations, in order to establish a continuous chain from the raw materials to the finished marketable product. Finally the economic reasons: an existing works with ample means and developed general services has at its disposal a much wider range of processes than a small and isolated unit with restricted means and limited capital investments.

Subject to this, and although such a classification may appear somewhat arbitrary, it would seem that the processes actually adopted in the industry and confirmed by experience, and which aim at giving the usual fertilisers a granulated form, can be grouped in three large categories.

(1) Spray Process.

This process consists of projecting the fertiliser in liquid form by means of a fine spray into the interior of a tower; the droplets fall freely through space, and condense under the impact of a counter-current of cold air. The product is collected at the bottom of the tower, and it is sufficient to screen it, to come within the limits laid down by the commercial guarantee.

(2) Process of flaking.

The liquid product is spread at the periphery of a rotating metal drum, cooled on the inside by a water jet. On contacting the cooled wall, crystallisation takes place, and the solid film formed is detached in flakes, crushed and screened according to the size required. It is also possible by another method to detach the film from the cylinder before complete crystallisation is achieved. Thus, fine flakes are obtained, which are introduced into a rotating cylinder while still in a plastic state. By rolling them in the interior of the apparatus, the flakes are made to assume a spherical shape, hardening as and when crystallisation takes place.

This latter process is to a certain extent a blending of the flaking and of the process of the rotating cylinder, which will have our closer attention later.

(3) Process of the rotating cylinder.

This is without doubt the most widely adopted process. It consists in the main of introducing to a rotating cylinder or "granulator" the finely divided raw material, and bringing about by some means an agglomeration within the moving mass. The controls in attaining this aim are temperature and moisture. The first of the two, aims at softening the fertiliser by semi-fusion and represents a delicate operation; the second is more widely adopted.

The principle consists in distributing in the mass a liquid, usually water, to which can be added a secondary product promoting granulation: sulphuric or phosphoric acid, silicates, diatomaceous earth etc., which act as binders.

Owing to the rotary movement of the cylinder, the particles constituting the mass roll over one another, gradually agglomerate and grow in size, finally leaving the cylinder as granules of more or less regular diameter, according to the success of the operation.

Of course, the water thus introduced must be eliminated afterwards, and the process includes, of necessity, a drying process. But the two phases, granulation and drying, need not be separate; they can be concomitant; the same apparatus (actually the drier) can act as granulator and drier. In the latter case the water, introduced at a given moment in the process, is eliminated as soon as it has played its rôle of supplying temporarily the necessary factor of cohesion. The elimination of this water must be slow, as a sudden drying may cause some products to disintegrate. Of course, it goes without saying that the presence, in the slurry to be granulated, of an element soluble in water constitutes a favourable factor. The quantity of water required per unit of material is inversely as the solubility of the latter. In outline, the combined operation of granulation and drying consists of introducing into a specially constructed drier a finely divided fertiliser, suitably moistened. (Moistening can take place during the process of sub-division previous to the drier), and of submitting the slurry to a rolling motion by rotation of the apparatus, a stream of hot gas being introduced at the same time to eliminate the water. At the exit of the drier the product is screened, both the fines and oversize particles - the latter after crushing - being returned to the cycle - this is called recycling.

The problem is complicated. In order to study it we have to formulate a certain number of hypotheses. Those hypotheses which commend themselves by dint of some probabilities should tend to simplify the problem. Starting from simple premises, we will endeavour to express it mathematically. The calculation will lead to a certain number of conclusions, which will be of value only if confirmed by experience. Experience having confirmed the laws which may have been formulated by reasoning, we propose to refrain from infering that the initial hypotheses were correct in every detail, but we are of the opinion that we will be justified in deducing some useful and practical information. That is the only purpose of this paper.

Chapter No. 1.

Basic hypotheses.

In what way does a powdered material rolling round in the interior of a rotating cylinder tend to agglomerate, so that fine particles grow progressively until they assume the size of marketable granules? Several answers may be given to this question.

It could be assumed that compact lumps form spontaneously and break up later, forming an infinite number of small elements with sharp corners, which are later rounded off by erosion. It could equally well be assumed that an optimum dimension of particles, brought about by an equilibrium of the disintegrating forces and forces of cohesion, corresponds for a given product to a given moisture content, a phenomenon related to those occasioned by forces of surface tension in liquids. But it would seem that the formation of compact blocks, which later are submitted to disintegration, only takes place, locally or accidentally, if the moisture is excessive. This case exists; it is rare, and is to be discarded if we limit ourselves to an ideal aspect of the question.

As to the second hypothesis, it postulates a quasi-spontaneous formation of granules. It is certainly possible to make observations of this kind. This point of view is corroborated by the presence of granules of an irregular geometrical form, but this irregular form originates from certain crystalline elements which are not sufficiently broken up during the previous crushing operations. In addition, it is in contradiction to the progressive evolution of the phenomenon, which has often been observed.

It could equally well be maintained in a third hypothesis that the definite volume of the granule is obtained by the successive amalgamation of juxtaposed elements. This last explanation seems to be most likely, and yet two different cases have to be distinguished. If it is assumed that very small elements "a" coalesce to form elements "2a" which again coalesce to form elements "4a" etc., in geometrical progression, it is possible to conceive that considerable dimensions are very rapidly attained, which, in our opinion, is not corroborated by experience. If, on the other hand, a simple arithmetical progression of the order "a" is substituted for the geometrical progression, we think we are nearer to reality. Although out of necessity we are getting away from this reality to a certain extent, we will finally retain the hypotheses of a progressive growth starting from a small element which, by constant rolling carries away by its movement, a small film of material at the expense of the powdery mass. We assume, in addition, that the thickness of this film of material remains proportionate to the residual moisture of the product. This idea of proportion expresses the fact that the moister the material, the greater its capacity of adherence. As drying progresses the pasteing of the film is reduced to become nil when the product is dry.

These two propositions are expressed as follows:

(1) a granule rolling like a bicycle wheel carries away through each rotation an adhering film of a certain thickness;

(2) The thickness of this film is proportionate to the moisture content at the moment.

Consider the case of a granule whose position in the drier in relation to the length of the drier (L) is denoted by x, which may therefore vary between 0 and L.

(See appendix No. 1 (a))

This granule is animated by two movements:

(1) a rotary movement due to the rotation of the apparatus.

(2) a slow progressive movement which tends to remove it from the apparatus.

The trajectory of the granule in the interior of the tube is in the form of a spiral.

If $\Delta \ell$ be the length of the spiral traversed during the time Δt, n being the number of rotations round its own axis during its travel $\Delta \ell$ of a granule of the diameter D the following is obtained:

$$n = \frac{\Delta \ell}{\pi D}$$

According to the basic hypothesis, the diameter "D" has augmented during the same time:

$$\Delta D = 2 n \mathcal{E}$$

$\mathcal{E}$ being the thickness of the film of the material carried away with each rotation of the granule.

In reality the following is obtained:

$\Delta \ell = n\pi (D + \mathcal{E})$ but $\mathcal{E}$ is assumed to be small compared with D.

From the two preceding equations the following may be deduced:

$$\Delta D = 2\mathcal{E} \frac{\Delta \ell}{\pi D}$$

The projection of the portion of the cylindrical spiral $\Delta \ell$ on the axis of the drier is of a length equal to Δx hence

$$\Delta \ell = \frac{R \omega}{V} \Delta x$$

with ω being the angular speed of the drier in radians per unit time.

V = speed of passage of granule from drier.
R = radius of cylinder.

These three values represent constants of construction and functioning of the drier.

The following is deduced:

$$\Delta D = \frac{2 \mathcal{E}}{\pi D} \frac{R \omega}{V} \Delta x$$

therefore

$$\boxed{D . dD = \frac{2R\omega}{\pi V} \mathcal{E}\, dx}$$

This equation we propose to call the "fundamental equation".

Its integration, which should lead to D = f (x), is complicated by the fact that $\mathcal{E}$ is a function of "x" and it is precisely the structure of this function which has to be determined.

Assuming that
W_p = moisture content of the product at the point x
W_a = moisture content of air " " " "

we shall have to determine:

$W_a = f(x)$
$W_p = f(W_a) = f(x)$
$\mathcal{E} = K W_p = f(x)$ and finally : $D = f(x)$.

General Determination of $W_a = f(x)$

Assuming that:
C_a = specific heat of the air in the drier $C_a = C^{te}$
C_v = average specific heat of water vapour in drier
k = coefficient of superficial convection
S = surface of contact of product with heated air

$$S = x \times \mathcal{E}_o$$

(See Appendix 1 (b))

r = average heat of evaporation of the water contained in the product

t' = temperature of the product assumed to be constant in the drier

t = temperature of air.

In contact with the surface dS of the material in the interior of the drier, the air used for drying shows a loss in temperature "dt" and the following may be written

$$(1) \quad (C_a + W_a C_v)\, dt = \pm\, k\, dS\, (t - t')$$

The sign of the second portion expresses the direction of circulation of the gas in relation to the product:

+ sign : rational circulation

- sign : parallel circulation.

Let us express, on the other hand, the fact that the calories given off by the air are utilised to evaporate the moisture of the product:

$$(2) \quad (C_a + W_a C_v)\, dt = -\, r . dW_a$$

It is possible to eliminate "t" between the two equations (1) and (2) and to obtain the relation $W_a = f\ (x)$.

Unfortunately the calculation ends with an integration of the form $y = \int \frac{dx}{\text{Log}\ x}$. This integral is eliptical and difficult to manipulate. With a sufficient approximation it is possible to write the following:

$$(1) \quad (C_a + W_{am} C_v)\, dt = \pm\, k' dx\, (t - t') \qquad \text{with } k' = klo$$

$$(2) \quad (C_a + W_{am} C_v)\, dt = -\, r . dW_a$$

W_{am} representing an average value of W_a.

The treatment is thus simplified, and we are induced to divide it into two separate chapters, according to whether the drying is effected in a parallel or rational fashion.

Chapter II

Rational Circulation

(A) DETERMINATION of $W_a = f\ (x)$

$$(3) \quad (C_a + W_{am} C_v)\, dt = +\, k'\, dx\, (t - t')$$

$$(4) \quad (C_a + W_{am} C_v)\, dt = r . dW_a$$

dW_a has constantly the opposite sign of dx and dt.

The elimination of "t" between (3) and (4) follows from a simple calculation on which it is needless to dwell, and which leads finally to 2 equations:

$$\lambda (W_a - W_{ao}) = (T - t')\, (C_a + W_{am} C_v)\, (1 - e^{q})$$

and (5) $$dW_a = \frac{k'}{r}\, (T - t')\, e^{-q} dx$$

$$\text{with } q = \frac{k'\ (x - L)}{C_a + W_{am} C_v}\ , \quad \text{or } \frac{k'\ (x - L)}{W}$$

when $W = C_a + W_{am} C_v$

and W_{ao} = value of W_a at the entry of the gas.

These expressions are the result of a logarithmic integration which can only be used if $(T - t')(C_a + W_{am}C_v) > r(W_a - W_{ao})$ an inequality verified a fortiori if in the second portion W_a is replaced by W_a W_a, representing the moisture of the air at the exit of the drier.

In order to verify the inequality

$(T - t')(C_a + W_{am}C_v) \geqslant r(W_{am} - W_{ao})$

T, the temperature of the gas on entering the drier, must have a value such, that the calories carried by the air entering the drier, are at least sufficient to eliminate all the moisture contained in the product in the case of an optimum rate of exchange (case where the air leaves at the temperature of the product).

(B) DETERMINATION OF $W_p = f(x)$

Let us assume that "a" kgs of air enter simultaneously with "p" kgs of product. The quantity of product corresponding to 1 kg of air $\frac{p}{a} = C^{te}$. The moisture W_p of the product decreases from the entry to the exit of the drier. A weight $\frac{p}{a}$ of a fertiliser loses in a unit of time "dt" a quantity of water equal to $\frac{p \cdot dW_p}{a}$ which enriches the corresponding air and the following is obtained:

$$dW_a = \frac{p}{a} dW_p$$

$$dW_p = \frac{-a k'}{p \; r}(T - t') e^{\varphi} dx$$

Integration is carried out without difficulty when observing that for x = L one has $W_p = H$, the initial moisture.

Finally, when all the calculations have been completed, the following is obtained:

$$(6) \quad W_p = H - N(T - t')(e^{\varphi'} - 1)$$

with $N = \frac{a}{rp} We^{-\varphi''}$

where $\varphi' = \frac{k'x}{W}$ and $\varphi'' = \frac{k'L}{W}$

It is, of course, necessary that the apparatus utilised, permits of a complete drying of the product, which implies that, for a length of drier L, the formula is as follows:

$$N(T - t')(e^{\varphi''} - 1) \geqslant H$$

(C) DETERMINATION OF: D = f (x)

We have assumed already that $\mathcal{E} = K W_p$

i.e. $\mathcal{E} = K\left[H - N(T - t')(e^{\varphi'} - 1)\right]$

and $D.dD = \frac{2\mathcal{E}}{\pi} \frac{R\omega}{V} dx$

i.e.

$$(7) \quad D.dD = 2\frac{R\omega K}{\pi V}\left[H - N(T - t')(e^{\varphi'} - 1)\right] dx$$

After completing calculation and assuming that at x = o one had D = o, i.e. that the initial mixture was of infinite fineness, integration results in an equation of the following form:

$$D^2 = \frac{4\ K\ R\ \omega}{\pi\ V} \left\{ \left[H + N\ (T - t') \right] x + N\ (T - t') \frac{W}{k'} (1 - e^{q'}) \right\}$$

or (8) $D^2 = A\ x + B - B\ e^{Mx}$

with $A = \frac{4}{\pi}\ \frac{R\ \omega K}{V}\ \left[H + N\ (T - t') \right]$

$B = \frac{4}{\pi}\ \frac{R\ \omega}{V}\ K\ N\ (T - t')\ \frac{W}{k'}$

$M = \frac{k'}{W}$

(D) DISCUSSION AND INTERPRETATION OF RESULTS.

(1) The second portion of the equation 7 which cancels itself out for a value of $x = x_o = \frac{W}{k'}\ \text{Log}\ (1 = \frac{H}{N\ (T - t')})$ shows that there exists in the drier a position corresponding to a point of a maximum diameter G_m and the equation 6 shows that this point is exactly the one where the moisture of the product becomes nil.

In practice it is obvious that this is somewhat different, and it must be assumed that the growth ceases or that its speed greatly diminishes starting from an abscissa $x_1 < x_o$.

With x_1 the speed of diffusion of the moisture of the granule becomes insufficient to ensure the surface formation of a moist film, responsible for the increase in volume.

(2) Construction of curve 8.

$$D^2 = Ax + B - Be^{Mx}$$

This is the difference between the ordinates to the right $Ax + B$ and the exponential Be^{Mx}; these two curves have in common the point $x = o$.

The examination of the origin of the tangents, the angular coefficients of which are A and M B respectively, indicate their relative position and one obtains the following plotting on which the striated portions show the evolution of the diameter of the granule.

(See Appendix No. 1 (c))

Note: The dotted portion of the curve represents the zone of retarded diffusion.

It goes without saying that the portion with the broken line corresponds to an imaginary portion.

Chapter "III".

(A) DETERMINATION of $W_a = f\ (x)$

(3)' $(C_a + W_{am}C_v)\ dt + - k'\ dx\ (t - t')$

(4)' $(C_a + W_{am}C_v)\ dt = - rdW_a$

dt is invariably of the opposite sign to dx and dW_a.

When operating as in a rational circulation the following two equations are obtained:

$$r(W_a - W_{ao}) = (T - t')\ (C_a + W_{am}\ C_v)\ (1 - e^{-q'})$$

(5) and $dW_a = \frac{k'}{r}\ (T - t')\ e^{-q'}dx$

Here again the integration postulates, as in rational flow that the inequality should be verified

$$(T - t')\ (C_a + W_{am}C_v) > r\ (W_a - W_{ao})$$

which can be replaced by the equation

$$(T - t') \quad (C_a + W_{am}C_v) \geqslant r\,(W_{aM} - W_{ao})$$

As has been seen, the latter inequality has always been verified by means of the adoption of the correct temperature of the entering gas.

(B) DETERMINATION of $W_p = f\,(x)$.

As with rational flow the following is obtained:

$$dW_a = -\frac{p}{a}\, dW_p$$

$$dW_p = -\frac{a}{p}\,\frac{k'}{r}\,(T - t')\, e^{q}dx$$

$$(6') \quad W_p = H - P\,(T - t')\,(1 - e^{-q})$$

with $P = \frac{a}{r\,p}\, W$

As with rational flow one encounters here the equation which should be satisfied by L and T

$$P(T - t')\,(1 - e^{-q''}) \geqslant H.$$

(C) DETERMINATION of $D = f\,(x)$

In following the same reasoning as in the preceding paragraph the following is obtained:

$$(7') \quad DdD = \frac{2\,R\,\omega}{\pi\,V}K\left[H - P\,(T - t')\,(1 - e^{-q'})\right]$$

By integration one obtains

$$(8) \quad D^2 = A'\,x + B' - B'\,e^{-Mx}$$

hence $D^2 = \frac{4\,K\,R\,\omega}{\pi V}\left\{\left[(H - P\,(T - t')\right]x + P(T-t')\,\frac{W}{k'}\,(1-e^{-q'})\right\}$

with A' and B' = Constants

$$A' = \frac{4\,K\,R\,\omega}{W}\left[H - P\,(T - t')\right]$$

$$B' = \frac{4\,K\,R\,\omega}{V}\,P\,(T - t')\,\left(\frac{W}{k'}\right) \text{ and } M = \frac{k'}{W}$$

(D) DISCUSSION AND INTEPRETATIONS OF RESULTS.

Let us note from now on that ,

$$H < P\,(T - t')$$

i.e.

$$H \leqslant \frac{a}{r\,p}\,(C_a + W_{am}\,C_v)\,(T - t')$$

inequality equivalent to that already encountered:

$$r\,(W_{am} - W_{ao}) \leqslant (C_a + W_{am}C_v)\,(T - t')$$

which expresses that by construction the calorific volume brought into play should be sufficient for the elimination of the moisture inherent in the product.

(1) The equation (6') shows that the moisture of the product becomes nil for a value of $x = x'_o$, so that:

$$e^{\frac{-k'\,x'_o}{W}} = 1 - \frac{H}{P\,(T - t')}$$

i.e. $x'_o = \frac{-W}{k'}\,\log\left(1 - \frac{H}{P\,(T - t')}\right)$

The abscissa $x = x'_o$ represents, as shown by equation (7') the maximum point of growth.

But as in rational circulation it must be assumed that in reality growth ceases or greatly diminishes its speed starting from an abscissa $x'_1 < x'_o$

CURVES OF W_p (Striated Portion)

(See Appendix 1 (d))

(2) Construction of curve (8') (See Appendix 1 (e))

$$D^2 = A' x + B' - B' e$$

One finds in this instance the same reasoning as in rational flow, the length of the striation showing the evolution of the actual growth.

Chapter "IV"

COMPARATIVE EVOLUTION OF MOISTURE OF THE PRODUCT IN THE RATIONAL & PARALLEL CASES

(A) COMPARISON OF VALUE W_p in the rational W_{pr} and in the parallel W_{pp}

(9) $W_{pr} = H - \lambda \bar{e}^{ML} (e^{Mx} - 1)$

(9') $W_{pp} = H - \lambda (1 - e^{Mx})$

$$\text{with } \lambda = \frac{a}{r\,p} (C_a + W_{am} C_v)(T - t')$$

$$M = \frac{k'}{C_a + W_{am} C_v}$$

For $x = o$ and $x = L$ the two values W_p are the same.

At the entrance of the drier $W_{pr} = W_{pp} = H$

At the exit $W_{pr} = W_{pp} = H - \lambda (1 - e^{-ML})$

In order that the moisture should be nil at the exit the length of the drier should correspond to the equation

$$\lambda (1 - e^{-ML}) = H$$

Hence $L \geqslant \frac{1}{M} \text{Log} \left(\frac{\lambda}{\lambda - H} \right)$ or, by way of clarifying

$$L \geqslant \frac{C_a + W_{am} C_v}{K\,eo} \text{Log} \frac{a(C_a + W_{am}C_v)(T - t')}{a(C_a + W_{am}C_v)(T - t') H r p}$$

(B) SLOPES OF THE TANGENTS OF THE TWO CURVES W_{pr} and W_{pp}

$$\text{The relation } \frac{dW_{pr}}{dW_{pp}} = e^{M(Zx-L)}$$

In the zone $x < \frac{L}{r}$ the parallel drying is the quickest.

At the point $\frac{L}{2}$ the thermic exchanges are identical.

In the zone $x > \frac{L}{2}$ it is the rational drying which is the most rapid.

(C). RELATIVE POSITIONS AT THE MAXIMUM POINT OF GROWTH OF GRANULES G_m.

In the rational it is possible by replacing N by Pe^{-ML} to write the following:

$$x_o = \frac{W}{k'} \text{Log} \left(1 + \frac{H}{\bar{P}(T - t') e^{-ML}}\right)$$

in parallel:

$$x'_0 = -\frac{W}{k'} \text{ Log } \left(1 - \frac{H}{P\,(T - t')}\right)$$

$$\text{with } H < P\ (T - t')$$

which is tantamount to saying that the abscissae of the two points x_0 and x'_0 are in the proportion of the logarithms of the numbers

for x_0 : $S = \frac{P\,(T - t')\,e^{-ML} + H}{P\,(T - t')\,e^{-ML}}$

for x'_0 : $S' = \frac{P\,(T - t')}{P\,(T - t') - H}$

The two numbers S and S' are equal if $H = P\ (T - t')\ (1 - e^{-ML})$

The two points x_0 and x'_0 of maximum growth in the rational and parallel merge. Their abscissa is as follows :-

$$x_0 = x'_0 = \frac{W}{k'} ML = L \text{ extreme end of drier.}$$

The comparison of the two derived expressions $\frac{dS}{dH}$ and $\frac{dS'}{dH}$ shows that if H diminishes to less than the value H_1, x'_0 decreases more rapidly than x_0: the point of maximum growth is therefore always much nearer the origin in parallel than in the rational.

The extreme case $H = P\ (T - t')$ is only obtained when $L = \infty$; then the following figure is obtained: -

(See appendix 1 (f))

which the following table expresses :-

H	$< P\ (T - t')\ (1 - e^{-ML})$	$= P\ (T - t')\ (1 - e^{-ML})$	$> P\ (T - t')$
X'_0	$< x_0$	$= x_0$	$= x_0 = x\infty$

The values W_{pr} and W_{pp} in function of x are represented by the difference between the ordinates of the horizontals H and those of the curves

$$\frac{W}{k'} \text{ Log } \left(1 + \frac{H}{P\,(T - t')\,e^{-ML}}\right)$$

and $-\frac{W}{k'} \text{ Log } \left(1 - \frac{H}{P\,(T - t')}\right)$

They are represented in the diagram (See Appendix No. 2 (g)) by striation, the first by a dotted line, the second by a broken line.

The two curves (1) and (2) represent the variation of the points of maximum growth in the rational and parallel resepctively, when the initial moisture H varies.

For a given value of "H" the corresponding points of maximum growth G_m are immediately obtained in x_0 and x'_0.

In the theoretical point L (farthest end of the drier) the two points merge; they can only be attained when the moisture at the start is equal to $P\ (T - t')\ (1 - e^{-ML})$.

C O N C L U S I O N S

Based on theoretical considerations, we have assumed right at the beginning that the rational and parallel systems were equivalent in regard to the thermic plan.

On this hypothesis the following conclusions are assumed:

(1) If H is greater than P (T - t') the drying has not been completed. This corroborates that which has been observed previously. This is a hypothesis which we have excluded.

(2) If H equals P (T - t') drying may be complete but it necessitates a drier of infinite length.

(3) If $H \leq P (T - t') (1 - e^{-ML})$ drying is complete.

(4) The point of maximum growth is farther down the tube in the rational than in the parallel and there exists an H value where the distance between the two points is at its greatest. These points merge at the exit of the drier for a value $H = P (T-t')(1 - e^{-ML})$

SPEED OF DIFFUSION

Let us suppose that, beginning with a moisture value of $i\,i' = i_1\,i'_1$ (see Appendix 2 (g)), the speed of diffusion is insufficient for W_p to obey the above law.

This value is reached in the drier in parallel flow well before it is reached in rational flow.

It may be assumed that from this point onwards, the drying is effected up to the exit proportionately to the duration of stay in the drier. The time spent in the drier is represented in parallel by the abscissa j L and in the rational by j_1 L.

Parallel drying is therefore undeniably superior.

Chapter " V "

COMPARATIVE EVOLUTION OF GROWTH ACCORDING TO WHETHER THE DRYING IS CARRIED OUT IN PARALLEL OR IN RATIONAL.

Although the preceding graphs give a sufficiently exact idea of the evolution of "D" it is of interest to give comparative plottings of the curves $D^2 = f(x)$ in the rational and parallel, as deduced from equations 8 and 8'.

$$(D^2)_r = Ax + B - Be^{Mx}$$
$$(D^2)_p = A'_x + B' - B'e^{-Mx}$$

$$\frac{d\,(D^2)r}{d\,(D^2)p} = \frac{A - MBe^{Mx}}{A' + MB'e^{-Mx}} = \frac{H - P\,e^{-ML}\,(T - t')\,(e^{Mx} - 1)}{H - P\,(T - t')\,(1 - e^{-Mx})}$$

The equation is invariably greater than 1 which implies that $(D^2)_r$ is at the beginning above $(D^2)_p$ and that it grows much more quickly whatever the value of "x".

The derived expressions $d(D^2)_r$ and $d(D^2)_p$ cancel each other out at the point G_m seen on the resepective abscissae x_o and x'_o with $x'_o < x_o$.

Finally, the following figure is obtained (See Appendix 2 (h)), the difference in diameter of the granules on leaving being represented by the segment o o'.

PARTICULAR CASE

One has seen that in the case of $H = P (T - t') (1 - e^{-ML})$ the points x_o and x'_o merge. The following figure is obtained: (See Appendix 2 (i)).

SPEED OF DIFFUSION

In order to take into account in actual practice the insufficiency of speed of the diffusion of moisture it has to be assumed that, as from a certain diameter D_o, the growth is not subject any more to the preceding mathematical laws, but to a linear law and the segment o o' becomes o_1 o'_1. In reality, calculation shows that the value of the segment o o' is as follows:

$$\frac{4\ K}{H} \quad \frac{R\omega}{V} \quad P\ (T - t')\ \left[Le^{-ML} + 2\ \frac{W}{k'}\ (e^{-ML} + 1)\right]$$

REMARK

It is possible to give the evolution of the size of granules a more striking form in observing that the two curves $D^2 = f\ (x)$ are represented by the difference of the ordinates of a straight line and of an exponential; the following is obtained:

(See Appendix 3 j)

There again the points x'_o and x_o and L are merged if $H = P\ (T - t')\ (1 - e^{-ML})$.

CONCLUSION

In parallel circulation the maximum point of growth is attained earlier and the size of the granule is much smaller than with the rational circulation.

CHAPTER "VI"

INFLUENCE OF THE VARIATION IN INITIAL MOISTURE "H" ON THE DRYING AND GRANULATION, RATIONAL AND PARALLEL HEATING.

(1) Action on the drying

Equations 9 and 9' give the following:

$$\frac{dW_{pr}}{dH} = \frac{dW_{pp}}{dH} = 1$$

At each point of the drier, moisture remains proportional to "H" whatever the system of drying adopted.

(2) Action on G_m final diameter of granules.

Equations 8 and 8' give the following:-

$$\frac{d\ (D^2)\ r}{dH} = \frac{d\ (D^2)\ p}{dH} = \frac{4\ K\ R\omega}{\pi\ V}\ Lf$$

Lf being the abscissa of the drier where the diameter is at its maximum.

CHAPTER "VII"

INFLUENCE OF THE VARIATION IN INITIAL TEMPERATURE OF DRYING GASES "T" ON THE DRYING AND GRANULATION IN RATIONAL AND PARALLEL CIRCULATIONS.

(1) Action on drying.

Equations 6 and 6' give the following:

$$W_{pr} = H - N\ (T - t')\ (e^{q'} - 1)$$

$$W_{pp} = H - P\ (T - t')\ (1 - e^{-q'})$$

As will be immediately seen, the following is invariably obtained:

$$dW_{pp} < o \text{ and } dW_{pr} < o\ .$$

Let us examine what becomes of the difference $dW_{pr} - dW_{pp}$ when x varies within the limits which interest us, i.e., o to L. The above quantity may be expressed as follows:

$$\left[\frac{K}{e^{Mx}} - e^{-ML}\ e^{2Mx} + e^{Mx}\ (1 - e^{-ML}) - 1\right]$$

$\frac{K}{e^{Mx}}$ is invariably positive.

The quantity in brackets is a trinome of the formula $a x^2 + bx + c$ which is cancelled out if

$$e^{Mx} = 1 \quad \text{i.e.} \quad x = o$$

$$e^{Mx} = e^{ML} \quad \text{i.e.} \quad x = L$$

In the interior of this segment i.e., all along the drier the trinome is positive and the following is obtained :

$$dW_{pr} - dW_{pp} > o$$

As both of these expressions are negative, the following happens:

$$|dW_{pr}| < |dW_{pp}|$$

This remark is important; it shows that any variation in "T" entails a corresponding variation in moisture content which is invariably greater if one works in parallel instead of rational circulation.

It is therefore much easier to make a correction by allowing an accidentally abnormal moisture to act on the temperature of the gases on entering in a parallel circulation than in a rational circulation.

(See Appendix 3 k)

II. ACTION ON THE FINAL DIAMETER.

By taking the equations 8 and 8' the following is obtained:

$$\frac{d(D_r^2)}{dT} = \frac{4\ KR\alpha}{\pi V}\left[Pxe^{-q''} + \frac{W}{k'}\ Pe^{-q''}(1 - e^{q'})\right]$$

$$\frac{d(D_p^2)}{dT} = \frac{4\ KR\alpha}{\pi V}\left[-Px + \frac{PW}{k'}(1 - e^{-q'})\right]$$

These two expressions are invariably negative, the portions in brackets being negative whatever the values of x; by putting $\frac{k'}{W} = M$ the following is obtained:

firstly: $\frac{P}{M}\ e^{-ML}\ (Mx + 1 - e^{Mx})$

When developed in a series the last factor gives the following:

$$-\frac{M^2x^2}{2!} - \frac{M^3x^3}{3!} - \frac{M^4x^4}{4!} \ldots\ \text{invariably negative;}$$

secondly: $\frac{P}{M}\ (-Mx + 1 - e^{-Mx})$. 1

and $\frac{P}{Me^{Mx}}\ (-Mx\ e^{Mx} + e^{Mx} - 1)$

The last factor of the last expression can also be written thus:

$$M^2x^2\left(\frac{1}{2} - \frac{1}{1!}\right) + M^3x^3\left(\frac{1}{3!} - \frac{1}{2!}\right) + M^4x^4\left(\frac{1}{4!} - \frac{1}{3!}\right)$$

it is always negative.

At each point of the drier the diameter varies in the inverse sense of the temperature at entry.

On the other hand, a variation of "T" influences all the more the size of the diameter of the granule, the farther the abscissa of the latter is from the mouth of the drier, which is evident, because at a given point this variation is briefly the integral of the preceding states submitted to the same effects.

Finally, let us examine the value of the difference:

$$d(D_r^2) - d(D_p^2)$$

This difference has the sign of the quantity:

$$(Mx + 1 - e^{Mx}) e^{-ML} - (-Mx + 1 - e^{-Mx})$$

or:

$$(Mx + 1 - e^{Mx}) + (Mx e^{Mx} - e^{Mx} + 1) e^{M(L - x)}$$

or:

$$X \quad + \quad Ye^{M(L-x)}$$

But $X = - \frac{M^2x^2}{2'} - \frac{M^3x^3}{3'} \dots\dots\dots < 0$

$$Y = M^2x^2 \left(\frac{1}{1} - \frac{1}{2'}\right) + M^3x^3 \left(\frac{1}{2'} - \frac{1}{3'}\right) + \dots\dots\dots > 0$$

As $X + Y = M^2x^2 \left(\frac{1}{1} - \frac{2}{2'}\right) + M^3x^3 \left(\frac{1}{2'} - \frac{2}{3'}\right) = \dots\dots\dots > 0$

one has a fortiori:

$$X + Ye^{M(L-x)} > 0 \quad , \quad e^{M(L-x)} \text{ being } > 1$$

It is possible to write the following:

$$d(D^2_R) > d(D^2p)$$

as each of these values $d(D^2_R)$ and $d(D^2_p)$ is negative an absolute value is obtained:

$$\left| \frac{d\, D^2\, R}{dT} \right| < \left| \frac{d\, (D^2\, p)}{dT} \right|$$

At some point of the drier any variation of T, the initial temperature of gases, has a greater influence on the diameter of a granule, if the circulation of the gases is carried out in parallel, than when it is carried out in rational circulation.

Chapter "VIII".

APPLICATION TO THE CONSTRUCTION OF A DRIER-GRANULATOR

At the end of these long and laborious discussions it seems to us necessary to recall the initial hypotheses and to draw up a balance sheet of the results obtained. On the behaviour of the materials we have uttered a certain number of postulates which should be used only with the greatest care. The method of granulation chosen as a premisefor the calculation is not unique and does not apply to all products. In our opinion this method commends itself by the greatest probability among all the types of processes which justify the method of the rotating drier. In addition, the exigencies of mathematical development have induced us to make a certain concession in regard to simplicity which is perhaps not altogether surprising. Contrary to reality, the calorific exchange is less simple than we had anticipated . It was necessary to allow for losses of calories by the radiation of the tube, for the heat of hydration of the solid phase, as well as for a certain number of heat transmission coefficients, among them a coefficient depending upon the relative speeds of gases and solids.

As to the physical implications, we have assumed a simple relation between the temperature and vapour tension provided that the heat given off by the gas served only and completely to vaporise the water contained in the liquid, and considering that the temperature of the product remained constant. In addition, the surface of the mass in the drier is neither even nor regular and the mass is certainly the seat of a number of secondary phenomena which escape analysis.

Finally, we have completely left out the well-known phenomenon of a hygroscopical material retaining a certain quantity of moisture in excess of that which is accounted for by the known laws of physics.

It is here that such a number of approximations is liable to vitiate the correctness of the results. Having assumed some rules, we have allowed reasoning to develop in the abstract without making any other conditions safe those which result from the actual development of the mathematical process. These conditions may be summarised by the following equations, which remain valid whatever the method adopted in regard to the circulation of gases:

$$H \leq \frac{a}{r\,p} \quad (C_a + W_{am}\,C_v)\ (T - t')\ (1 - e^{-ML})$$

or

$$L \geqslant \frac{C_a + W_{am}\,C_v}{keo}\ \text{Log.}\ \frac{a\,(C_a + W_{am}C_v)\quad (T - t')}{a\,(C_a + W_{am}C_v)\quad (T - t') - H\,r\,p}$$

(The equality $H = \frac{a}{r\,p} \quad \frac{C_a + W_{am}C_v}{keo}\ (T - t')$ would require a drier of infinite length).

But we have given an interpretation of these relations and shown that they correspond to the logical aspect of matters. They express a mathematical relationship between the length of the drier, the initial moisture of the product and the minimum calorific quantity to be supplied to the system. Let us hasten to say that they express only an approximate aspect of the evolution of sizes in relation to each other, and that it would be a great mistake to admit them as such, and without discernment for the calculation of an apparatus. Above all, the homogeneity of the equations would have to be discussed. On the other hand, one cannot fail to be astonished at the influence exercised by the factor "T".

It seems, in fact, to follow from preceding reasonings that the temperature of the gases used for drying affects granulation in the same way as moisture: in other words, an increase in the moisture "H" can be compensated by an increase in the temperature "T". This, it appears, is going against the generally admitted fact which an English paper published towards 1938, specially and judiciously underlined. It is certain that, in the control of granulation, a slight increase in moisture can be compensated by a slight decrease in temperature and this artificial device is well known by operators and frequently used by them.

But, on the one hand, it should be noted that the term "T" was always affected by the coefficient $\frac{a}{p}$ $(C_a + W_{am}C_v)$, and all that has been said of "T" applies in reality to a calorific quantity $(C_a + W_{am}C_v)\ T = Q$, which implies that the variations of H should, in reality, be offset by an additional supply of calories, and there should be no possible dispute.

On the other hand, the artificial device of using the special term $\frac{dT}{dH} < o$ is in reality a useful transitory means which it is unsafe to use for practical purposes when pursuing an operation.

It has, in fact, been established that moisture gives an impulse to granulation and that is correct. It is further correct that the size of granule increases when H increases. If one corrects an excessive value of H by lowering of T, whatever the immediate advantages to be gained, one is hopelessly tied up with a relentless cycle. All the recycled out-size portion returns to a circuit which is charged with an increasing moisture, the whole is increasingly thrown out of gear, the size of granules becomes disproportionate, the re-establishment of granulation, an extremely unstable system, becomes completely impossible. A process which, at a given instant is suitable, has generally speaking more drawbacks than advantages. After making these points, the results obtained may be condensed into the following few remarks, some of which may appear in the guise of evidence, but the merit of which is to find a definite justification in practical experience:

(1) If H equals P (T - t') complete drying is only assured if $L = \infty$; it becomes impossible if $H > P\ (T - t')$. In reality it is necessary that $H \leq P\ (T - t')\ (1 - e^{-ML})$

(2) The point of maximum growth is much more rapidly attained in parallel than in rational flow, the distance between these two points is known and a function of H and T; if H and T satisfy the equation $H = P\ (T - t')\ (1 - e^{-ML})$ they merge.

(3) The maximum diameter of a granule is larger in the rational than in parallel flow, the difference between the two diameters being proportional to T and inversely proportional to the exponential of L.

(4) The action on granulation of a variation in initial moisture content is the same in both systems.

(5) The moisture content of a product at a given point of the drier is a decreasing function of the temperature at the entrance of the drier, but the incidence of a variation of T on the moisture at a given point is greater in the parallel than in the rational system.

(6) The size of granules varies in the inverse ratio of calories supplied, and this variation is increasing when travelling along the length of the tube in the direction in which the material advances, the culminating point of this variation being situated at a point where the diameter itself ceases to grow, owing to the lack of residuary moisture.

Finally, the variation in the maximum diameter at a given point, due to a variation of Q, is more accentuated in the parallel than in the rational.

(7) Finally, only from the point of view of drying, it may be of interest to heat in parallel on account of the slow speed of diffusion of moisture.

The conclusions should permit the choice of a method of circulation for the granulation of a given product, once the aptitude of granulation of the product is known, the limit of temperature which it can stand and the moisture which it requires. They outline the path to follow for the planning of a granulating drier. One will begin by determining by way of experiments the duration of drying of a granule marketable in size, which gives the time the material has to remain in the apparatus. Taking into account production capacity, the charge to be dealt with is deduced.

From this will be deduced the volume of the charge in motion, in other words, the volume of the drier, noting that the rate of charging must in normal circumstances not be greater than 20%, a percentage which has often been admitted. Preliminary experiments will have determined H, T, and the method of circulation.

As a second step, the total quantity of calories to be brought into play to eliminate all the moisture introduced, will have to be determined; this calorific quantity has to be calculated on a liberal basis, taking into account losses by radiation; in this determination it will be necessary to consider that the gases on leaving the drier should have a temperature somewhat higher than the dew-point in order to avoid condensation before entering the atmosphere; finally, especially if parallel circulation has been adopted, it is necessary that the vapour tension of the water in the gas at the exit should be much lower than the equilibrium tension of the product, in order to avoid with certainty any taking up of moisture in the last stages of travel.

T being known, the volume of the gas to be introduced can be deduced therefrom immediately, as well as the fuel consumption required to give it adequate calorific capacity; by the same process the characteristics of the ventilator to be adopted will be determined.

Finally, the diameter of the drier should be such that the average speed of the gases is compatible with the density of fine particles of the product to avoid excessive carry-over and facilitate the working conditions of the cyclone situated below the apparatus. The length L follows automatically.

Of course, as always in such cases, inconsistencies may occur in the course of calculation, and compromises will often have to be resorted to.

The interior shape of the apparatus must conform to the double rôle of granulator and drier and the question of fittings must be studied very closely. These fittings must render possible an evacuation of the mass in the time required for the duration of stay in the drier; the forward movement in the interior of the apparatus can be arranged by internal fittings or by inclining the tube. In our opinion it is preferable for technical reasons to attain the forward movement by inclining the interior fittings rather than an inclination of the tube itself, but the latter solution is always possible. The determination of the incline is a fortiori a delicate matter. At present Etablissements Kuhlmannare studying this question, and are endeavouring to establish a formula which can be readily applied. On the other hand, formulae are actually available which one can use with greater or less success. The problem is therefore almost solved, and it remains for us to examine, in order to bring this to a close, some rules which are likely to be applied for a rational control of the apparatus.

Chapter "IX"

REGULATION OF GRANULATION

The carrying out of granulation is relatively easy; the maintenance of this state is much more difficult owing to its instability. The slightest mishap becomes worse if it is not immediately remedied, and a state of disturbance prevails which requires much care and time to overcome. In addition, if the corrective measures are not applied with care, the whole system may be affected with a stoppage of production. We have up to now established that granulation evolved as a function of the initial moisture H, provided that this moisture is eliminated according to a predetermined law. On the other hand, we have recognised the possibility of connecting the initial temperature of gases with H by way of a second parameter; but we have also shown that the action on T could come into play in the opposite direction to the one we are seeking, and included more drawbacks than advantages. By maintaining T at its optimum value by an appropriate regulation, the influence of this factor will be eliminated.

On the other hand, we have recognised the advantage of using instead of T the parameter $Q = \frac{a\,(C_a + W_{am}C_v)\,(T)}{p}$

which represents, briefly speaking, the number of calories introduced per unit of the mass of the product. It would, therefore, be possible to correct H by Q i.e., to vary the volume of the gas introduced into the apparatus at a fixed temperature T as a function of the initial moisture H.

The regulation of Q is reduced to the checking of a gas flow at a constant temperature: this is a known and simple problem. The instantaneous determination of H can also be relatively simple; it can be assumed that, before moistening, the moisture content of the raw materials is constant, and everything would depend on the extent of the flow of the liquid. But the problem is more complicated when one has to deal with a going industrial installation which, in addition to the raw materials introduced, must absorb the fines and the crushed oversize, coming from the drier after elimination of the product of proper size.

Then again, the recycling varies very much as to quantity, and its intrinsic moisture content is not constant. It should be moistened to the same extent as the raw materials with which it is being mixed. In view of the fact that the moisture content of the recycled material is unknown, one is forced to adjust,with a certain margin of error, the initial moisture H. Owing to recycling there exists already a cause for error, which must not be ignored. In addition, if one were to ignore this cause of error, this problem would be only solved at the entrance of the tube. As a matter of fact one has to take into account the previous history of the mass in the apparatus which very likely is not in a state of dryness commensurate with the conditions at the entrance. In other words, at an instant, when the water and the calories introduced are regulated by a harmonious relationship, the moisture of the moving mass is not correlated to the initial conditions, the calories introduced are either insufficient or excessive, and the point of regulation is difficult to attain or will be attained only after a considerable lapse of time. In order to surmount these difficulties, we will choose Q as regulation parameter and not the moisture at the entrance but at the exit. It so happens that the temperature of the gases at the exit of the drier is exactly representative of both the volume of the actual water introduced and also of the state of interior dryness of the system.

The optimum temperature of the gases at the exit of the tube is definitely determined by experience, the difference between the actual temperature of the gases and the optimum temperature serving to favour the impetus of the gas flow at the mouth, which is maintained at an absolutely constant temperature.

Two regulations are carried out concurrently:

(1) Regulation of T, corresponding to a fixed value, regulating the flow of the fuel.

(2) Control of volume of gas entering at the exit temperature. Such a device has given in actual practice satisfactory results. It obviously does not embrace the whole of the problem in all its complexity, but what device could ever cope with this!

CONCLUSIONS

In the course of this study we have endeavoured to establish some of the basic principles governing granulation. We have limited ourselves to one particular case which appears to us the most general. Certain processes, though carried out in the rotary drier, evade these principles, and it is necessary to seek rules which direct the process of their evolution in fields different from those which we have tried to explore; this, among other things, is the case of granulation by semi-fusion. In the course of this study we have recalled on several occasions the liberties which we had to take with reality, and the concession we had to make to mathematical developments. In drafting this text we have often asked ourselves whether we have not given way to a mirage of fiction, and if we could be reproached with having purely and simply attained an abstraction. We will not take up a categorical attitude towards this point, but we think we are entitled to recall the care with which we have limited the field of application of the mathematical results obtained. Many of the conclusions arrived at elsewhere have been forshadowed in detail before being arrived at by calculation, and after having been attained by calculation they have been confirmed by experience. May we be permitted to add that the long and at times dry pages should not constitute a case for this or that method of production or for this or that type of apparatus. We have,as a matter of fact, completely and purposely neglected the economic aspect of the problems. On the other hand, the series of processes applicable to a given product may be extremely restricted, and this will rarely give rise to hesitation. But then that is another question.

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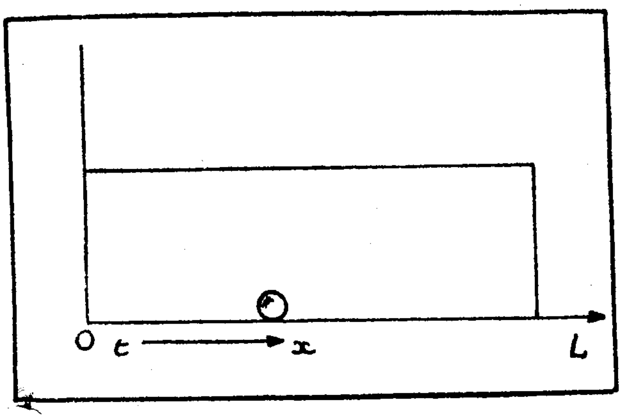
ANNEXE No.1.

Appendix

(a)



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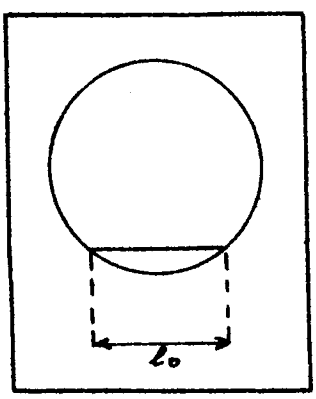

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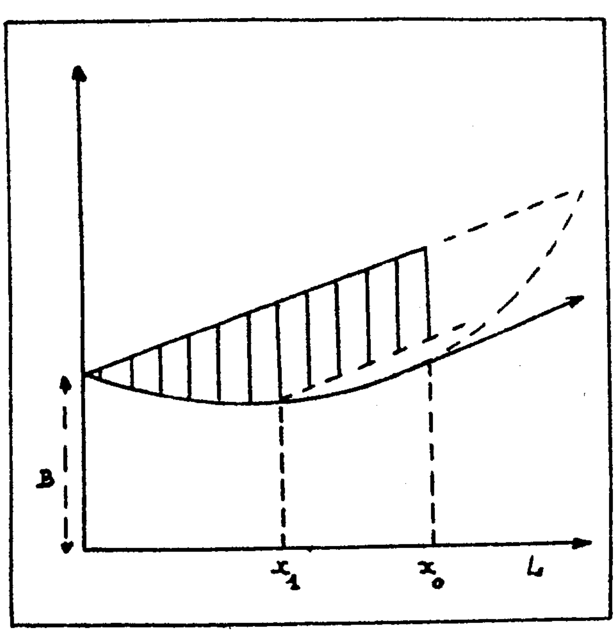

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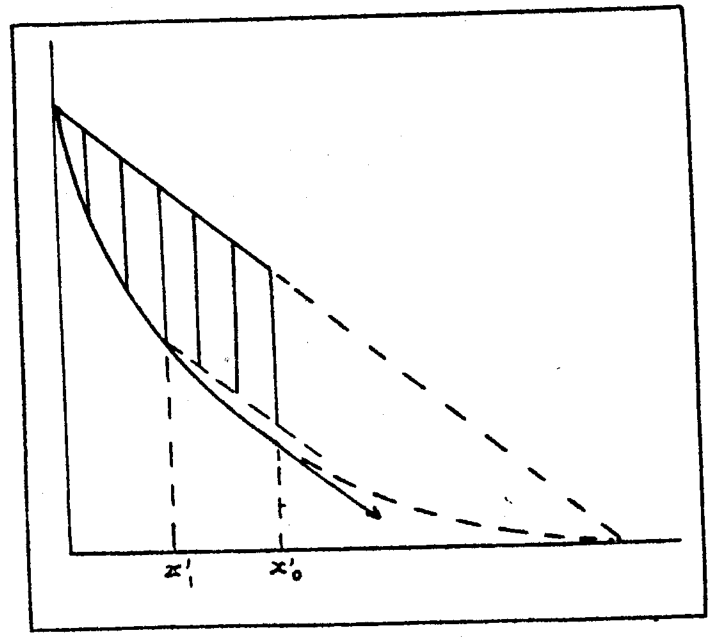

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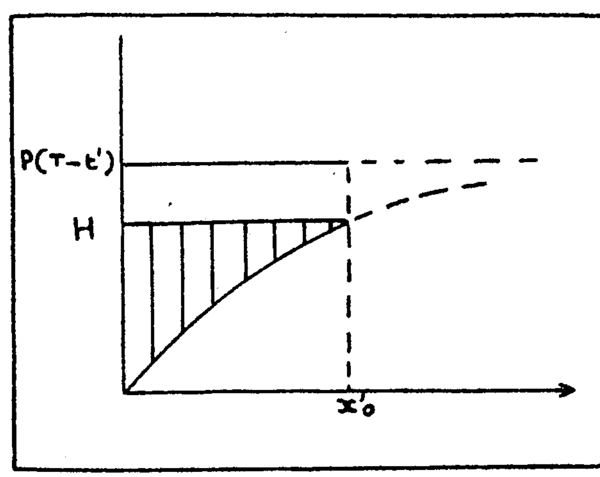

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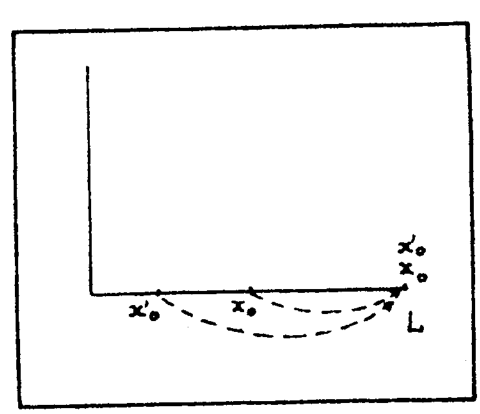
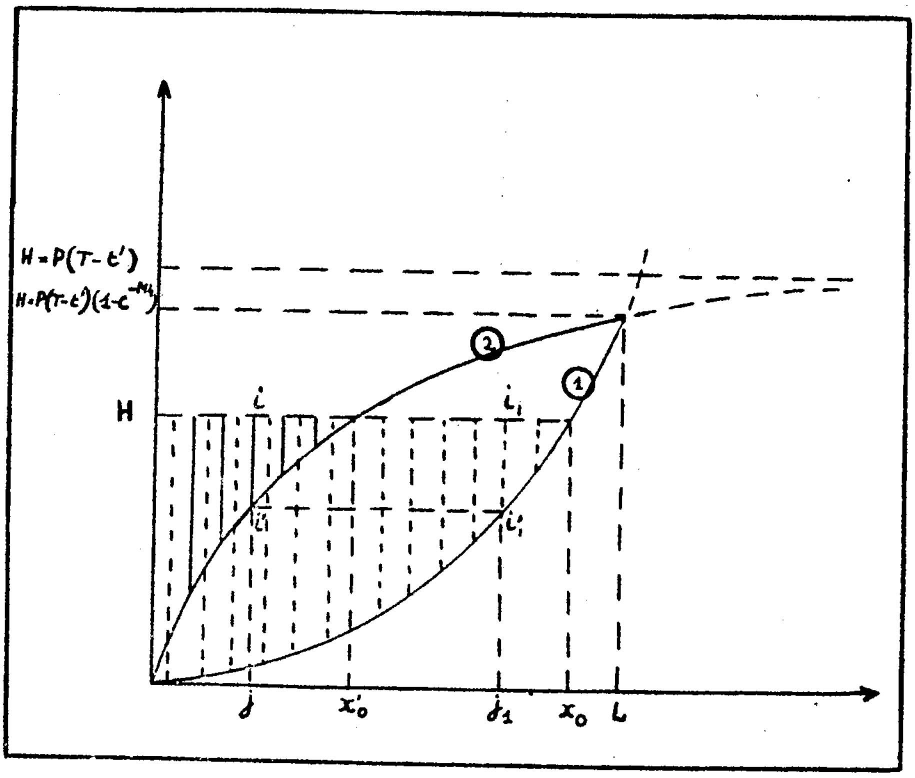


ANNEXE No.2

Appendix

(g)



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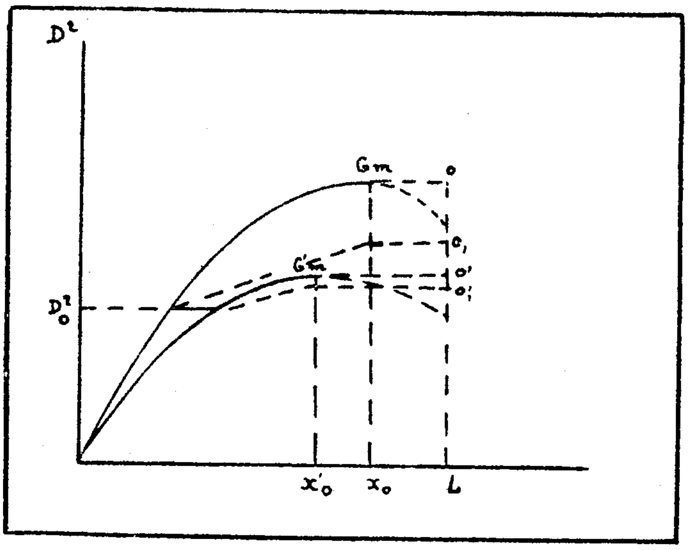

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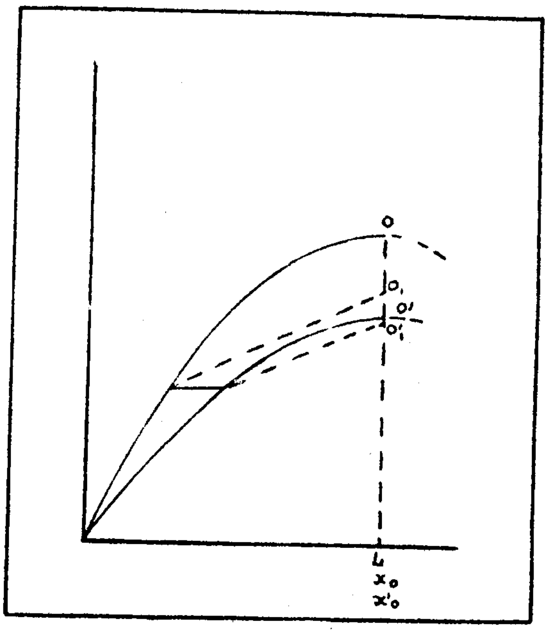
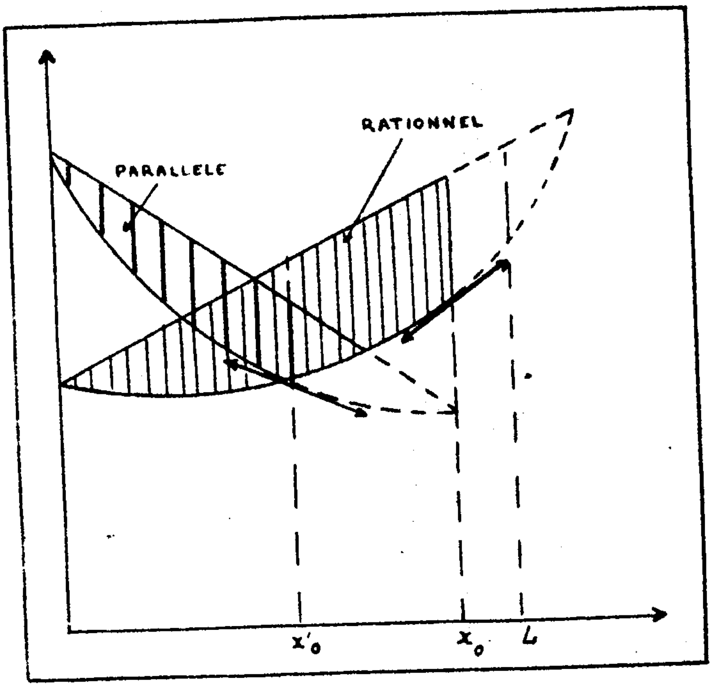


ANNEXE No.3

Appendix

(j)



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